

11回目レポート課題

番号： 名前：

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回答欄が足りない場合はレポート用紙に書いて、この用紙とともにステープラーで留めること

1 次の1階微分方程式を解きなさい。ただし、 x は t の関数 $x(t)$ とし、 $x' = dx/dt$ とする。

(1) $x' - 3x = 0, x(0) = 2$

解 $x(t) = 2e^{3t}$

(2) $x' - 2x = e^{2t}, x(0) = 3$

解 $x(t) = (t + 3)e^{2t}$

(3) $x' - x = 2 \sin t, x(0) = 1$

解 $x(t) = 2e^t - \sin t - \cos t$

(4) $x' - 2x = 0, x(0) = 3$

解 $x(t) = 3e^{2t}$

(5) $x' + x = e^{-t}, x(0) = 1$

解 $x(t) = (1 + t)e^{-t}$

(6) $x' + 3x = 2e^{-t}, x(0) = 1$

解 $x(t) = e^{-t}$

(7) $x' + 2x = e^{-3t}, x(0) = 3$

解 $x(t) = 4e^{-2t} - e^{-3t}$

(8) $x' - 2x = 5 \cos t, x(0) = 2$

解 $x(t) = 4e^{2t} + \sin t - 2 \cos t$

(9) $x' - x = t, x(0) = 2$

解 $x(t) = -1 - t + 3e^t$

(10) $x' - 2x = t, x(0) = 4$

解 $x(t) = -\frac{1}{4} - \frac{1}{2}t + \frac{17}{4}e^{2t}$

(11) $x' + x = e^{2t}, x(0) = 1$

解 $x(t) = \frac{2}{3}e^{-t} + \frac{1}{3}e^{2t}$

(12) $x' - 3x = e^t, x(0) = 1$

解 $x(t) = -\frac{1}{2}e^t + \frac{3}{2}e^{3t}$

(13) $x' + 3x = 2 \sin 3t, x(0) = 1$

解 $x(t) = \frac{4}{3}e^{-3t} + \frac{1}{3} \sin 3t - \frac{1}{3} \cos 3t$

(14) $x' - x = e^{2t}, x(0) = -2$

解 $x(t) = e^{2t} - 3e^t$

(15) $x' + 2x = 6t - 1, x(0) = 3$

解 $x(t) = 5e^{-2t} + 3t - 2$

(16) $x' - 2x = 0, x(0) = 3$

解 $x(t) = 3e^{2t}$

(17) $x' + 3x = 4e^t, x(0) = 1$

解 $x(t) = e^t$

(18) $x' - x = 2 \cos t, x(0) = 3$

解 $x(t) = 4e^t - \cos t + \sin t$

(19) $x' + x = U(t - 2), x(0) = 1$

解 $x(t) = e^{-t} + (1 - e^{-(t-2)})U(t - 2)$

[問題は裏面に続く]

2 次の2階微分方程式を解け.

(1) $x'' + x' - 2x = 12t, x(0) = 0, x'(0) = 6$

解 $x(t) = -3 - 6t + 6e^t - 3e^{-2t}$

(2) $x'' - 4x = \cos t, x(0) = 0, x'(0) = 0$

解 $x(t) = -\frac{1}{5} \cos t + \frac{1}{10} e^{-2t} + \frac{1}{10} e^{2t}$

(3) $x'' + x' - 2x = 3e^t, x(0) = 1, x'(0) = -1$

解 $x(t) = e^{-2t} + te^t$

(4) $x'' - 3x = 1, x(0) = 1, x'(0) = 1$

解 $x(t) = -\frac{1}{3} + \frac{4 + \sqrt{3}}{6} e^{\sqrt{3}t} + \frac{4 - \sqrt{3}}{6} e^{-\sqrt{3}t}$

(5) $x'' + 2x' + 5x = 0, x(0) = 1, x'(0) = 0$

解 $x(t) = \frac{1}{2} e^{-t} (2 \cos 2t + \sin 2t)$

(6) $x'' + 2x' - 3x = e^t, x(0) = 1, x'(0) = 0$

解 $x(t) = \frac{1}{16} (5e^{-3t} + 11e^t + 4te^t)$

(7) $x'' + 4x' + 4x = \sin 2t, x(0) = 0, x'(0) = 1$

解 $x(t) = \frac{1}{16} (e^{-2t} + 10te^{-2t} - \cos 2t)$

(8) $x'' + x' - 2x = -12e^t, x(0) = 1, x'(0) = 0$

解 $x(t) = -e^{-2t} + 2e^t - 4te^t$

(9) $x'' - x = t, x(0) = 1, x'(0) = 4$

解 $x(t) = -t + 3e^t - 2e^{-t}$

(10) $x'' + 2x' = 5 \cos t, x(0) = 0, x'(0) = 2$

解 $x(t) = 1 - \cos t + 2 \sin t$

(11) $x'' - x' - 6x = 0, x(0) = 1, x'(0) = 1$

解 $x(t) = \frac{3}{5} e^{3t} + \frac{2}{5} e^{-2t}$

(12) $x'' - 3x' + 2x = e^t, x(0) = 0, x'(0) = 0$

解 $x(t) = -te^t - e^t + e^{2t}$

(13) $x'' - 4x' + 4x = 2e^{2t}, x(0) = 0, x'(0) = 1$

解 $x(t) = te^{2t} + t^2 e^{2t}$

(14) $x'' + 4x = 8e^{2t}, x(0) = 0, x'(0) = 0$

解 $x(t) = -\cos 2t - \sin 2t + e^{2t}$

(15) $x'' - 2x = 2, x(0) = 0, x'(0) = 3$

解 $x(t) = -2 - t + 2e^{2t}$

(16) $x'' - 4x' + 4x = 0, x(0) = 1, x'(0) = 1$

解 $x(t) = e^{2t}(1 - t)$

(17) $x'' + 6x' + 10x = 0, x(0) = 1, x'(0) = 1$

解 $x(t) = e^{-3t}(\cos t + 4 \sin t)$

(18) $x'' + x = 2 \sin t, x(0) = 2, x'(0) = 4$

解 $x(t) = 5 \sin t + (2 - t) \cos t$

(19) $x'' + 2x' + x = t, x(0) = 0, x'(0) = 0$

解 $x(t) = (t + 2)e^{-t} + t - 2$

(20) $x'' - 3x' + 2x = U(t-3), x(0) = 1, x'(0) = 0$

解 $x(t) = 2e^{2t} - e^t + \frac{1}{2} (1 - 2e^{t-3} + e^{2(t-3)}) U(t-3)$

(21) $x'' + 4x' - 5x = 0, x(0) = 0, x'(0) = 1$

解 $x(t) = \frac{1}{6} (e^t - e^{-5t})$

(22) $x'' - 6x' + 9x = 0, x(0) = 1, x'(0) = 2$

解 $x(t) = (1 - t)e^{3t}$

(23) $x'' - 4x' + 13x = 0, x(0) = 2, x'(0) = 1$

解 $x(t) = e^{2t}(\sin 3t + 2 \cos 3t)$

(24) $x'' + 3x' - 4x = 6e^{2t}, x(0) = 6, x'(0) = 2$

解 $x(t) = 4e^t + e^{-4t} + e^{2t}$

(25) $x'' + 2x' + 2x = 10 \sin 2t,$
 $x(0) = -1, x'(0) = -3$

解 $x(t) = e^{-t} \cos t - 2 \cos 2t - \sin 2t$

(26) $x'' - 2x' + x = te^t, x(0) = 0, x'(0) = 0$

解 $x(t) = \frac{t^3 e^t}{6}$